International Journal of Innovation and Modern Applied Science

Vol. 11 Issue 2 , December 2025

Doi : https://doi.org/10.57109/ra25

ISSN No.: 665-8984

Early Damage Detection in Civil Engineering Structures: A Regularized Inverse Problem Framework

Abdellah Riyahi [1], Mohammed Mestari [1], Bouchra Bouihi [1]*

[1] *2IACS Laboratory, ENSET Mohammedia, Hassan II University of Casablanca, Mohammedia, Morocco*

**Corresponding author: a.riyahi@enset-media.ac.ma*

ABSTRACT: Although early damage detection in civil engineering structures is difficult, because damage is not directly observed, yet it is key for structural health monitoring and predictive maintenance. It must be inferred from partial, noisy, and indirect vibration measurements. In this paper, a theoretical framework in which early damage detection is formulated as an improperly posed inverse problem has been developed. Uniqueness, stability, and practical recoverability are limited by weak damage, sparse sensing, environmental variability, and poor conditioning. Addressing these challenges requires recasting damage inference as a regularized problem, balancing data consistency and prior structural knowledge. The interpretation of smoothness, sparsity, bounded variation, physics-based, and probabilistic regularization principles is conducted in relation to different damage morphologies. Also, the paper distinguishes detectable, identifiable, estimable, and practically recoverable damage, as well as discussing the relevance of this framework for data-driven, physics-informed, and maintenance-oriented Structural Health Monitoring.

Keywords: Structural Health Monitoring; inverse problems; early damage detection; regularization; predictive maintenance; vibration-based diagnosis; civil engineering structures; recoverability.

1. INTRODUCTION

Bridges, buildings, viaducts, and other strategic infrastructure systems are among civil engineering structures that are continuously exposed to aging, fatigue, environmental stressors, operational loads, as well as progressive material deterioration. In this context, Structural Health Monitoring (SHM) has gained its importance as a component of infrastructure management; this is due to its support in the anticipation of adverse performance evolution and the assessment of structural condition. Its high practical value especially poses itself when structural changes are identifiable at an early stage, that is, before damage develops into severe serviceability loss, expensive repair, or safety-critical degradation. Thus, from the perspective of predictive maintenance, early damage detection is not only a diagnostic convenience, but also a prerequisite for rational and opportune intervention planning (Hou & Xia, 2021; Tibaduiza Burgos et al., 2020).

Not being readily measured, structural damage leads early damage detection to be a scientific challenge. Constitutive degradation, stiffness losses, or cracks are not detectable by vibration-based monitoring systems. Alternatively, they resort to recording structural responses: strains, accelerations, displacements, or derived dynamic characteristics, among others. Accordingly, the primary scientific challenge is inferential; that is, hidden structural changes must be reconstructed, given indirect and insufficient observations of structural response. Sparse sensor layouts, noise contamination, model uncertainty, unknown or poorly defined excitation, and operational and environmental unpredictability add to the complication of this inference. In reality, benign variations introduced by temperature, traffic, humidity, or shifting boundary conditions may be similar to faint damage signals in terms of amplitude (Cianci et al., 2026; Hou & Xia, 2021).

Consequently, these challenges imply that for early damage detection to be identified, the mathematical language of inverse problems should be utilized. In the direct context, the dynamic response of the structure is affected by loading, boundary conditions, material characteristics, and structural parameters. In the inverse context, on the contrary, measurable structural responses are utilized as an attempt to infer unknown structural parameters, or more precisely, damage-related disturbances; which is a fundamental reversal of direction. SHM seeks to reconstruct hidden causes from observed effects, while the structural system maps hidden causes to observable effects. This point of view is compatible with the more general claim that many structural engineering problems can be seen as inverse problems, and as such, they could profit from the conceptual and analytical tool created in inverse problem theory (Gallet et al., 2022).

In the early-damage regime, the inverse problem theory becomes more obviously applicable. When damage is still in its early stages, it usually has little impact on global structural observables and may only manifest as a slight change in modal properties, stiffness, or damping. The simple invertible mapping from data to damage is typically prohibited under such a framework. By generating comparable measured responses, various internal damage situations may result in non-uniqueness. Instability can result from little data perturbations causing significant changes in the estimated damage. Furthermore, limited resolution and incomplete identifiability may result from insufficient observations to ensure meaningful recovery of the actual damage distribution. Thus, in the Hadamard sense, the state of early damage inference being ill-posed is frequent: this could be due to existence being model-dependent, uniqueness failing under incomplete observation, and stability being severely compromised by noise and confounding variability (Gallet et al., 2022; Lehký et al., 2023).

According to this perspective, regularization is a mathematical requirement rather than a secondary numerical device. After structural damage detection is identified as an ill-posed inverse problem, further data that can stabilize the solution process is needed for meaningful inference. Smoothness assumptions, sparsity hypotheses, bounded-variation principles, constitutive admissibility restrictions, temporal monotonicity, probabilistic prior distributions are examples of such information. Regularization limits the class of acceptable solutions in each of these situations, making the recovered damage state more stable, physically comprehensible, and useful for engineering decision-making (Gallet et al., 2022, 2022).

Damage indicators, statistical diagnosis, model updating, and more recently data-driven and physics-informed approaches are among subjects addressed by extensive work in the literature of Structural Health Monitoring. However, the mathematical status of early damage inference often remains implicit. This is true. Numerous studies design detection algorithms or predictive models, yet fewer works examine in a unified manner whether the targeted damage is actually detectable, uniquely identifiable, stably estimable, or practically recoverable under realistic sensing limitations. The role of regularization is, similarly, often embedded in optimization procedures, machine learning architectures, or prior modeling choices. Although, it is not elevated to a central theoretical principle. An important conceptual gap results from this. High empirical performance does not lead automatically to stable structural recoverability; a theoretically existing damage state may remain practically unrecoverable under partial and noisy measurements, if no appropriate regularization or prior structural knowledge is introduced (Gallet et al., 2022; Tibaduiza Burgos et al., 2020).

This paper addresses that gap by developing a theoretical framework in which early damage detection in civil engineering structures is formulated explicitly as a regularized inverse problem. The paper interprets structural dynamics as the direct problem linking structural parameters to measurable response quantities. It then formulates damage inference as the inverse recovery of latent damage variables from vibration-based observations. Within this framework, early damage is analyzed as a weak-perturbation regime in which observability is limited and the inverse map is typically ill-posed. The paper further argues that regularization provides the mathematical mechanism through which unstable inversion may be converted into a stable and physically meaningful surrogate inference problem. Being essential for trustworthy SHM and predictive maintenance, particular emphasis is placed on the distinction between detectable, identifiable, estimable, and practically recoverable damage. In addition, the framework clarifies how modern data-driven and physics-informed models may be interpreted as approximations or constrained surrogates of inverse mappings rather than as mechanisms that bypass ill-posedness altogether. This theoretical direction is consistent with recent vibration-based and physics-informed SHM developments, including our recent work on data-driven and physics-informed neural networks for the Z24 bridge benchmark (Riyahi, 2025).

The remainder of the paper is organized as follows. Section 2 formulates the theoretical framework by introducing the direct problem in structural dynamics, the inverse formulation of damage detection, the weak-perturbation regime of early damage, the ill-posedness of damage inference, the role of regularization, and the taxonomy of recoverability. Section 3 discusses the implications of this framework for modern data-driven and physics-informed SHM as well as for predictive maintenance of civil engineering structures. Section 4 concludes the paper.

2. MATERIALS AND METHODS

2.1. Direct problem in structural dynamics and observation model

To rigorously formulate structural damage detection as an inverse problem, it is necessary to first define the corresponding direct problem. In vibration-based SHM, the direct problem consists in predicting the measurable structural response from a given set of physical parameters describing the structure. These parameters typically include mass distribution, stiffness properties, damping characteristics, and boundary conditions, which together define the dynamical behavior of the system (Gallet et al., 2022; Lehký et al., 2023)(Gallet et al., 2022; Lehký et al., 2023).

Let $\Omega \subset \mathbb{R}^d$denote the structural domain and let $\theta \in \Theta$denote the vector or field of structural parameters, where $\Theta$is the structural parameter space. The structural state is described by a time-dependent field $u(t) \in X$, where $X$is an appropriate state space, typically a Hilbert space representing displacement, velocity, or acceleration fields.

A general form of structural dynamics equations can be written as

$$\mathcal{M}(\theta)\ddot{u}(t) + \mathcal{C}(\theta)\dot{u}(t) + \mathcal{K}(\theta)u(t) = f(t), t \in (0, T),$$

where $\mathcal{M}(\theta)$, $\mathcal{C}(\theta)$, and $\mathcal{K}(\theta)$ denote the mass, damping, and stiffness operators, respectively, and $f(t)$ denotes the external excitation.

In practice, the full state $u(t)$ is not observable. Instead, measurements are obtained through an observation operator $\mathcal{O}: X \to Y$, where Y is the measurement space. The measured data are therefore written as

$$y(t) = \mathcal{O}(u(t)) + \eta(t),$$

where $\eta(t)$ represents measurement noise and modeling discrepancies.

By combining structural dynamics with the observation process, one defines the forward operator

$$F: \Theta \to Y, F(\theta) = \mathcal{O}(u_\theta),$$

where u_θ is the solution associated with parameter θ. This operator encapsulates the physical chain linking internal structural properties to observable data.

In many SHM applications, the observation operator is highly restrictive. Sensors may be sparsely distributed, measuring only selected degrees of freedom, often as accelerations at a limited number of locations. As a result, the measurement space is significantly lower dimensional than the structural state space, and a large portion of the structural information is not directly accessible. Furthermore, the excitation $f(t)$ is often not fully known, especially in ambient vibration monitoring, where wind, traffic, or environmental actions act as partially unknown inputs. More generally, one may write

$$y = F(\theta, \xi) + \eta,$$

where ξ denotes nuisance variables such as environmental and operational conditions. This already indicates that the corresponding inverse problem will be affected by indirectness, incompleteness, and confounding effects (Hou & Xia, 2021; Cianci et al., 2026).

2.2. Damage detection as an inverse problem

Let $\theta_0 \in \Theta$ denote a reference parameter corresponding to the undamaged or nominally healthy structure. Let $d \in D$ denote a damage variable, where D is the damage space. Depending on the modeling scale, d may represent local stiffness reductions, distributed constitutive perturbations, or parameter losses in a finite element representation (Lehký et al., 2023; Gallet et al., 2022).

The damaged state may then be written in the generic form

$$\theta = \theta_0 + d,$$

or, under a loss convention,

$$\theta = \theta_0 - d, d \geq 0.$$

The exact convention is secondary provided that it is used consistently. What matters is that $d = 0$ corresponds to the reference state and $d \neq 0$ represents structural alteration.

Using the forward operator F, one defines a damage-to-data map

$$G: D \to Y, G(d) = F(\theta_0 + d).$$

The observation model becomes

$$y^\delta = G(d^\dagger) + \eta, \| \eta \|_Y \leq \delta,$$

where $d^\dagger \in D$denotes the true but unknown damage state.

The inverse problem is therefore:

$$\text{Given } y^\delta\text{, recover } d^\dagger \text{ such that } y^\delta \approx G(d^\dagger).$$

In spite of being purposefully general, this formulation encompasses scenarios in which y^δ comprises modal features, raw time signals, frequency-domain quantities, or engineered indicators. These are derived from measured responses. While the data constitute only indirect observations of that change, the unknown is a hidden structural change.

Additionally, distinguishing between parameter estimation, state estimation, and model inference is crucial. Early damage detection, in Structural Health Monitoring, usually unites model inference in addition to parameter estimation. Reconstructing the dynamic state and deducing latent parameter variations or structural inconsistencies, that are perceived as damage, are the main objectives. It is not automatically implied, via a good estimate of the response, that it is a good estimate of the damage pattern that caused it (Gallet et al., 2022).

2.3. Early damage as a small perturbation and low-sensitivity regime

Usually, early damage does not appear as a large and abrupt structural modification. Frequently, it emerges gradually through localized microcracking, connection loosening, fatigue-related degradation, or even small stiffness losses. This regime is naturally modeled as a small perturbation of a reference structural state (Hou & Xia, 2021).

Let

$$\theta = \theta_0 + \varepsilon h, 0 < \varepsilon \ll 1,$$

where $h \in \Theta$defines the perturbation pattern and εits amplitude. If the forward operator $F$is Fréchet differentiable at θ_0, then

$$F(\theta_0 + \varepsilon h) = F(\theta_0) + \varepsilon F'(\theta_0)[h] + o(\varepsilon).$$

Hence, the perturbation in the data satisfies

$$\Delta y := F(\theta_0 + \varepsilon h) - F(\theta_0) \approx \varepsilon F'(\theta_0)[h].$$

This relation shows that the observable signature of early damage scales linearly with its amplitude at first order. When $\varepsilon$is small, the induced variation in the data may be extremely weak, possibly of the same order as measurement noise or environmental variability.

The derivative $F'(\theta_0)$captures the sensitivity of the measured data to infinitesimal perturbations of structural parameters. If $F'(\theta_0)[h]$is small or nearly vanishing for a given pattern h, then the corresponding damage configuration is weakly observable. Thus, detectability depends not only on damage amplitude, but also on the alignment between the damage pattern and the sensitivity structure of the forward operator.

A similar interpretation appears in modal analysis. Under small perturbations of stiffness, first-order modal perturbation suggests

$$\Delta\lambda_i \approx \langle \phi_i, \Delta\mathcal{K}\phi_i \rangle,$$

where λ_i and ϕ_i denote eigenvalues and mode shapes, respectively. If damage occurs in a region with low modal strain energy for a given mode, the resulting modal shift may be negligible. This explains why certain localized damage scenarios may remain nearly invisible in low-order modal features (Hou & Xia, 2021; Tibaduiza Burgos et al., 2020).

2.4. Ill-posedness of early damage inference

Once structural damage detection is formulated as an inverse problem, the next question is whether the problem is mathematically well posed. In the classical Hadamard sense, a problem is well posed if a solution exists, is unique, and depends continuously on the data. Early damage inference in civil engineering structures typically violates at least one, and often all, of these requirements (Gallet et al., 2022; Lehký et al., 2023).

The first possible failure concerns existence. There may be no exact $d \in D$ such that $G(d) = y^{\delta}$, because measured data may fall outside the range of the idealized forward operator. This may happen due to model simplifications, unknown boundary conditions, omitted environmental effects, or uncertain excitation.

The second failure concerns uniqueness. The operator G need not be injective on the admissible damage space. There may exist distinct damage states $d_1 \neq d_2$ such that

$$G(d_1) = G(d_2),$$

or, more realistically,

$$\| G(d_1) - G(d_2) \|_Y \ll \| d_1 - d_2 \|_D.$$

From this, we can deduce that various internal structural scenarios probably generate nearly or exactly the same observations. Not only sparse sensing and low modal sensitivity act against uniqueness, but also parameter coupling, limited bandwidth, and structural symmetries all do the same (Gallet et al., 2022; Hou & Xia, 2021).

Stability is the main concern of the third failure. Despite the existence and uniqueness of a solution, the inverse map to depend continuously on the data is not guaranteed. An estimate of the form is a target,

$$\| d_1 - d_2 \|_D \leq C \| G(d_1) - G(d_2) \|_Y,$$

but such an inequality is generally false, or at least very weak in inverse problems of structural damage detection.

A useful way to expose instability is through the linearized inverse problem around the healthy state. If G is Fréchet differentiable, then

$$G(d) \approx G(0) + G'(0)[d].$$

Setting $A := G'(0)$and $b^{\delta} := y^{\delta} - G(0)$, the linearized problem becomes

$$Ad \approx b^{\delta}.$$

If A is compact, smoothing, or poorly conditioned, its singular values decay toward zero. In a singular-value decomposition viewpoint, inverse recovery involves division by small singular values, which amplifies noise dramatically. In SHM terms, weakly observable damage directions correspond to near-null directions of the linearized forward operator, and inference along those directions is highly unstable (Gallet et al., 2022).

Partial observability serves to further intensify Ill-posedness; and then major portions of structural information remain unobserved, if the measurement process samples only a few response channels or locations. There is a possibility that damage patterns concentrated in poorly instrumented regions lie close to the kernel, or near-kernel, of the effective observation process.

Difficulty comprises a number of layers. One of these layers concerns environmental and operational variability. In the model

$$y = G(d, \xi) + \eta,$$

temperature, humidity, traffic, changing supports, or other non-damage effects may be represented by nuisance variables ξ. If enough information to separate d from ξ is not contained by the data, the inverse problem, then, becomes not only ill-posed but also partially non-identifiable (Cianci et al., 2026).

2.5. Regularization as the mathematical remedy

Because direct inversion is unstable or meaningless, early damage detection must be reformulated as a regularized inverse problem. From a variational viewpoint, one seeks

$$\hat{d}_{\alpha}^{\delta} \in \arg\min_{d \in \mathcal{A}} \left[\mathcal{D}(G(d), y^{\delta}) + \alpha \mathcal{R}(d) \right],$$

where $\mathcal{D}$is a data-fidelity term, $\mathcal{R}$is a regularization functional, $\alpha > 0$is a regularization parameter, and $\mathcal{A} \subset D$is an admissible set reflecting hard constraints (Gallet et al., 2022; Chen et al., 2020).

The data-fidelity term measures consistency between the predicted structural response and the observed measurements. A common choice is

$$\mathcal{D}(G(d), y^{\delta}) = \| G(d) - y^{\delta} \|_Y^2.$$

The regularization term encodes prior structural knowledge. Without such a term, the inverse problem admits a large number of unstable or physically meaningless solutions. Regularization therefore acts as a selection principle.

A classical choice is Tikhonov regularization:

$$\mathcal{R}(d) = \| d - d_{\text{ref}} \|_D^2,$$

which penalizes large deviations from a reference state, typically $d_{\text{ref}} = 0$. This is especially suitable when early damage is expected to remain small.

If damage is expected to be localized, sparsity-promoting regularization is more appropriate. Sparse Bayesian and sparsity-based formulations have been shown to be especially meaningful for structural damage identification under limited measurements (Chen et al., 2020). In such cases,

$$\mathcal{R}(d) = \| d \|_1$$

or a related sparse prior favors solutions where only a small number of components of $d$are nonzero, which aligns with isolated cracks or localized stiffness reductions.

Regularization becomes relevant, in case damage is represented as a field and predicted to demonstrate bounded-variation, total-variation, or sharp transitions:

$$\mathcal{R}(d) = \mathrm{TV}(d).$$

Even while suppressing spurious oscillations, gradual smooth solutions with discontinuities are granted.

Not only gradual smooth solutions are allowed, but physics-based constraints are also possibly introduced, such as

$$d(x) \geq 0$$

for stiffness loss, or temporal monotonicity constraints

$$d_{t+1}(x) \geq d_t(x).$$

These constraints may define the admissible set or enter the regularization functional as penalty terms.

A probabilistic interpretation is obtained in the Bayesian framework. If $\pi(d)$denotes a prior on the damage variable and $\pi(y^\delta \mid d)$the likelihood, then

$$\pi(d \mid y^\delta) \propto \pi(y^\delta \mid d)\pi(d),$$

and maximum a posteriori estimation yields a regularized optimization problem in which the negative log-prior plays the role of the regularization term (Gallet et al., 2022).

Regularization improves existence, stability, and partial uniqueness, but it introduces bias. This trade-off is fundamental: stronger regularization yields more stability but may oversmooth or attenuate true damage, whereas weaker regularization preserves detail but increases instability. The appropriate choice depends on the expected morphology of damage and on the informational quality of the measurements.

2.6. Recoverability: detectable, identifiable, estimable, and practically recoverable damage

A central conceptual contribution of this paper is the distinction between four notions that are often conflated in SHM.

Detectable damage is the weakest notion. A damage state $d^\dagger$is detectable relative to the healthy state if the damaged and undamaged responses are sufficiently separated in measurement space:

$$\| G(d^\dagger) - G(0) \|_Y > \tau,$$

where $\tau$is a threshold determined by noise, nuisance variability, and decision tolerance.

Identifiable damage is a stronger notion. A damage state is identifiable in an admissible class $\mathcal{A} \subset D$if

$$G(d) = G(d^{\dagger}), d \in \mathcal{A} \Rightarrow d = d^{\dagger}.$$

Thus, identifiability is an injectivity property of the forward map on a relevant structural class.

Estimable damage requires stability. A damage state is estimable if there exists a regularized estimator

$$\mathcal{E}_{\alpha}^{\delta}: Y \to D, \hat{d}_{\alpha}^{\delta} = \mathcal{E}_{\alpha}^{\delta}(y^{\delta}),$$

that depends in a controlled way on data perturbations and yields a meaningful approximation of $d^{\dagger}$.

Practically recoverable damage is the strongest and most engineering-relevant notion. A damage state is practically recoverable if the associated estimate is sufficiently stable, interpretable, and decision-relevant to support inspection or maintenance action at the required engineering scale.

These notions satisfy the logical chain

practical recoverability ⇒ estimability ⇒ identifiability ⇒ detectability,

while the converses generally fail. A damage state may be detectable without being identifiable, identifiable in principle without being estimable under realistic noise, and estimable without being sufficiently informative for real maintenance action. This hierarchy is especially important for early damage detection, where the limiting factor is often not raw anomaly sensitivity but stable inverse recoverability (Gallet et al., 2022; Lehký et al., 2023)

3. RESULTS AND DISCUSSION

3.1. Theoretical significance of the inverse-problem perspective

Early damage detection is far from being only a pattern-recognition task; it is rather considered as a latent-variable inference problem, under severe informational constraints. This is exhibited in the framework developed in Section 2. Measured response variation and inferred structural alteration are explicitly distinguished by the shift in perspective, and that is the reason of its importance. The hidden structural state has probably produced the signal itself. Thus, it is the true object of interest in vibration-based Structural Health Monitoring (Gallet et al., 2022).

Also, conceptual overstatements are prevented by this perspective. Therefore, there is no equivalence between a change in frequency, damping, or response statistics and a reliable damage estimate. Whether the available data actually contain sufficient information to support uniqueness, stability, and structural interpretability is firmly related to the inference problem being formulated explicitly.

3.2. Why early damage is especially difficult

The weak-perturbation analysis shows that early damage is not simply a smaller version of severe damage. It is a qualitatively different regime in which the observable signature may be of the same order as noise and environmental variability. As a consequence, detectability thresholds emerge naturally.

For instance, if

$$\| \boldsymbol{F}(\boldsymbol{\theta_0} + \boldsymbol{\varepsilon h}) - \boldsymbol{F}(\boldsymbol{\theta_0}) \|_{\boldsymbol{Y}}$$

is not significantly larger than the combined effect of measurement noise and nuisance variability, this means that the data do not support reliable distinction between damaged states and healthy states. Hence, a damage state probably remains practically unrecoverable despite being physically real. This can be considered as one reason, among others, behind environmental effects being a central challenge in vibration-based Structural Health Monitoring (Cianci et al., 2026; Hou & Xia, 2021).

3.3. Regularization as a structural necessity

Regularization is not only a numerical convenience, but also a structural necessity. This is a major theoretical implication posed by the analysis. In general, ill-posedness of the inverse problem implies that direct inversion cannot be trusted. Still, regularization presents stabilizing prior information. The original problem is accordingly transformed into a meaningful surrogate inference problem.

Distributed degradation is best suited by smoothness-based regularization, namely corrosion or diffuse material aging. Likewise, localized cracking or isolated joint deterioration are favorably well-set with sparsity-based regularization. When gradual smooth damage fields with sharp transitions are expected, they offer suitable conditions for total-variation regularization. Adding to that, constraints which are physics-based, as well as Bayesian priors, further enrich the admissible solution class. This proves that different regularization philosophies and different engineering assumptions are corresponded to each other (Gallet et al., 2022, 2022).

The choice of regularization therefore has both mathematical and physical meaning. It shapes the notion of recoverability and directly influences the interpretation of SHM outputs.

3.4. Implications for data-driven and physics-informed SHM

The inverse-problem framework also clarifies the role of modern artificial intelligence methods in SHM. A data-driven model can be written abstractly as

$$\boldsymbol{\mathcal{N}}_{\boldsymbol{\psi}}: \boldsymbol{Y} \rightarrow \boldsymbol{D},$$

where the learned mapping attempts to reconstruct damage-related quantities from measured data. In this sense, machine learning models are approximations of inverse maps (Yuan et al., 2020).

Learned mappings cannot be reliable unless some form of regularization is present; this happens if the inverse problem is ill-posed. This regularization is often more implicit practically. It arises from architectural constraints, training procedures, dataset bias, or penalty terms. Thus, data-driven models do not eliminate ill-posedness; they address it implicitly.

Thanks to physics-informed models, this connection is clearer. That is, when the learning objective incorporates physical laws, the end model acts as a regularized inverse inference mechanism, which is constrained by structural knowledge. As a result, interpretability is improved and robustness may be enhanced. Yet, the fundamental limits associated with sparse sensing, weak sensitivity, and environmental confounding are not eliminated.

This whole interpretation is strengthened by recent studies on physics-guided and physics-informed damage identification. Showing that robustness under data scarcity can be improved by embedding governing equations or structural priors further proves the interpretation. Meanwhile, recoverability is still dependent on measurement informativeness as well as modeling choices (Guo & Fang, 2023; Miele et al., 2023; Wang et al., 2022).

This interpretation is directly relevant to hybrid SHM approaches, including our previous work on the Z24 bridge benchmark, where data-driven and physics-informed models were used for vibration-based damage diagnosis (Riyahi et al., 2026). The present paper extends that line of work by clarifying the mathematical conditions under which such approaches can be trusted. It is also consistent with broader recent reviews on the transition from traditional damage detection toward physics-informed machine learning in bridge monitoring (Mammeri et al., 2025).

3.5. Implications for predictive maintenance of civil structures

The proposed framework has direct implications for predictive maintenance. Maintenance decisions should not be based solely on anomaly detection in measurement space, but on structurally meaningful and stability-aware damage inference.

A first implication concerns the transition from anomaly indication to damage interpretation. A detectable deviation may justify increased vigilance, but not necessarily immediate intervention. A stably estimable and practically recoverable damage pattern may justify targeted inspection or prioritized maintenance.

A second implication concerns sensor strategy. Because recoverability depends strongly on the observation operator, sensor placement should be designed not only for coverage but for inverse informativeness. Sensors should be positioned so as to maximize sensitivity to target damage modes and reduce observational equivalence between competing damage scenarios (Gallet et al., 2022).

A third implication concerns uncertainty-aware decision-making. Regularization stabilizes inference but does not eliminate uncertainty. Therefore, maintenance planning should rely on estimates whose uncertainty, spatial ambiguity, and decision-scale adequacy are explicitly considered.

Finally, benchmark structures such as instrumented bridges or buildings should not be treated only as datasets for algorithm comparison, but as conceptual testbeds in which detectability, identifiability, and recoverability can be examined under controlled conditions. This approach allows theory and application to interact without reducing the contribution to a purely experimental study. In that respect, the connection between digital twins, physics-based models, and machine learning is especially promising for maintenance-oriented SHM, provided that inverse identifiability and uncertainty remain central concerns (Cianci et al., 2026; Ritto & Rochinha, 2021).

4. CONCLUSION

Not only the major inferences of the work are clearly explained in this conclusion, but its importance and relevance are also foregrounded.

The theoretical framework that this paper has developed aims to formulate early damage detection in civil engineering structures, in relation to SHM, as a regularized inverse problem. The lack of a direct approach to observe structural damage, in vibration-based SHM, is the core argument in this paper. It is, on the contrary, an obligation that it is inferred from indirect, incomplete, and noisy measurements of structural response. For the problem to be within the scope of inverse problem theory, it is imperative to make this inferential character explicit (Gallet et al., 2022).

Measurement limitations, environmental variability, and poor conditioning strongly affect the corresponding inverse map. Also, it is generally ill-posed and weakly informative in the small-damage regime. All this leads early damage detection to be fundamentally difficult, which is what the paper has demonstrated. Regularization is not regarded as an optional computational device for transforming unstable inversion into stable and physically meaningful surrogate inference; this requires regularization to be seen a foundational mathematical mechanism (Gallet et al., 2022; Lehký et al., 2023).

A further contribution of the paper lies in the distinction between detectable, identifiable, estimable, and practically recoverable damage. This conceptual hierarchy clarifies that the existence of a structural perturbation does not guarantee its reliable recovery from available measurements. A damage state may be detectable without being identifiable, identifiable without being stably estimable, and estimable without being sufficiently robust or interpretable for maintenance action.

A possible interpretation of modern data-driven and physics-informed SHM approaches is offered, as well, by the framework through a rigorous lens. Bypassing inverse-problem difficulties is no longer an option. Rather, such approaches would be better acknowledged as explicit or implicit regularized inference mechanisms, whose reliability is not only dependent on predictive performance, but also on the informativeness of the data, the adequacy of the embedded priors, and the degree to which the learned mapping aligns with the true forward physics and the recoverability limits of the monitored structure (Guo & Fang, 2023; Riyahi, 2025; Wang et al., 2022).

From a predictive maintenance standpoint, a more trustworthy and transparent use of Structural Health Monitoring outputs is supported by the proposed theory, encouraging the design of monitoring systems. These systems are not only sensitive to anomalies, but also informative with respect to structurally meaningful damage variables. In civil engineering, actionable structural knowledge and signal variation are brought closer to each other by means of the inverse-problem.

Future work may extend this framework toward optimal sensor placement, stochastic inverse formulations that better separate damage from environmental variability, and conceptual validations on well-instrumented infrastructure benchmarks. Such developments would further strengthen the connection between inverse problem theory, structural health monitoring, and predictive maintenance.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the 2IACS Laboratory, ENSET Mohammedia, Hassan II University of Casablanca, for providing a stimulating academic and research environment for this work.

The authors would like to acknowledge the use of Grammarly for proofreading and improving the clarity of the manuscript.

REFERENCES

Cianci, E., Civera, M., De Biagi, V., & Chiaia, B. (2026). Physics-informed machine learning for the structural health monitoring and early warning of a long highway viaduct with displacement transducers. *Mech. Syst. Signal Process.*, *242*(113659), 113659.

Gallet, A., Rigby, S., Tallman, T. N., Kong, X., Hajirasouliha, I., Liew, A., Liu, D., Chen, L., Hauptmann, A., & Smyl, D. (2022). *Structural engineering from an inverse problems perspective*. *478*(2257), 20210526.

Guo, X.-Y., & Fang, S.-E. (2023). Structural parameter identification using physics-informed neural networks. *Measurement (Lond.)*, *220*(113334), 113334.

Hou, R., & Xia, Y. (2021). Review on the new development of vibration-based damage identification for civil engineering structures: 2010–2019. *Journal of Sound and Vibration*, *491*(115741), 115741.

Lehký, D., Šplíchal, B., Lamperová, K., & Slowik, O. (2023). Inverse analysis and optimization-based model updating for structural damage detection. *Ce/Pap.*, *6*(5), 1228–1233. https://doi.org/10.1002/cepa.2136

Mammeri, S., Barros, B., Conde-Carnero, B., & Riveiro, B. (2025). From traditional damage detection methods to Physics-Informed Machine Learning in bridges: A review. *Engineering Structures*, *330*(119862), 119862.

Miele, S., Karve, P., & Mahadevan, S. (2023). Multi-fidelity physics-informed machine learning for probabilistic damage diagnosis. *Reliability Engineering & System Safety*, *235*(109243), 109243.

Ritto, T. G., & Rochinha, F. A. (2021). Digital twin, physics-based model, and machine learning applied to damage detection in structures. *Mech. Syst. Signal Process.*, *155*(107614), 107614.

Riyahi, A. (2025). *Z24 Bridge NN & PINN Source Code Repository*. https://github.com/AbdellahRiyahi/Z24_Bridge_NN_PINN.git

Tibaduiza Burgos, D. A., Gomez Vargas, R. C., Pedraza, C., Agis, D., & Pozo, F. (2020). Damage Identification in Structural Health Monitoring: A Brief Review from its Implementation to the Use of Data-Driven Applications. *Sensors*, *20*(3), Article 3. https://doi.org/10.3390/s20030733

Wang, Z., Zhang, Y., & Chen, X. (2022). WaveNet-based vibration analysis for structural damage detection. *Mechanical Systems and Signal Processing*, *165*, 108299. https://doi.org/10.1016/j.ymssp.2021.108299